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# Comparison of $J_C$ in GdBCO tape using Dc magnetisation and harmonic Ac susceptibility measurements.

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**Abstract**—We have measured the critical current density ( $J_C$ ) of a sample of SuNAM HTS tape using Ac Susceptibility (ACS) and Dc Magnetisation (DCM) techniques. In DCM measurements, inhomogeneity in the applied Dc field causes a systematic underestimate of  $J_C$ . The error in DCM measurements is characterized by the penetration parameter,  $\gamma = \frac{3\pi\chi_0\delta B}{(3w - \frac{w^2}{l})\mu_0 J_C}$ , and grows as  $J_C$  decreases. Using a harmonic ACS analysis, we have obtained more accurate measurements of  $J_C$  as a function of applied Dc field and temperature for  $\gamma \approx 1$ .

## I. INTRODUCTION

Dc magnetisation is often used to determine the critical current density in superconducting materials. This method of characterization is very sensitive to inhomogeneity in the applied Dc field [1]. For values of the penetration parameter  $\gamma < 1$ , the field inhomogeneity ( $\delta B$ ) causes the magnetic field profile in part of the superconductor to reverse over the course of a DCM measurement, and results in a systematic underestimate of  $J_C$ . For  $\gamma \geq 1$ , the field inhomogeneity is sufficiently large to entirely reverse the field profile in the superconductor and  $J_C$  will appear to go to zero. In this case, DCM measurements of  $J_C$  are not reliable. We have used a Quantum Design PPMS to perform DCM and ACS measurements of  $J_C$  on a  $4 \times 5.5$  mm sample of SuNAM high temperature superconducting (HTS) tape as a function of applied Dc field and temperature.

The magnetic response of the tape was analysed using Bean's critical state model [2] with a spatially independent  $J_C$ , which leads to the well established 'rooftop' internal field distribution that has been confirmed experimentally using Hall probe microscopy [3]. For ACS measurements, analytic expressions for Ac susceptibility,  $\chi = \chi' + i\chi''$ , were used [4]. **For the remainder of this paper, Ac methods will be taken as the default unless otherwise stated.**

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## II. CALCULATIONS

For the analysis in this paper, we used Bean's critical state model [2] to relate the magnetic response of a superconducting sample to a field independent  $J_C$  using one of Maxwell's equations,

$$\nabla \times \mathbf{B} = \pm \mu_0 J_C \hat{\mathbf{j}}, \quad (1)$$

and the definition of magnetic moment,

$$\mathbf{m} = \frac{1}{2} \int \mathbf{r} \times \mathbf{J} dV. \quad (2)$$

### A. Dc Magnetisation

For a rectangular film of dimensions  $l \times w \times t$ , with  $l > w \gg t$  in a transverse field, the magnetic moment can be found by simple integration across the surface of the sample where

$$\mathbf{m} = \frac{t}{2} \left[ \int_{-\frac{w}{2}}^0 \int_{\frac{w-l}{2}+x}^{\frac{l-w}{2}-x} x J_C dy dx + \int_0^{\frac{w}{2}} \int_{\frac{w-l}{2}-x}^{\frac{l-w}{2}+x} -x J_C dy dx + \int_{\frac{l}{2}}^{\frac{l}{2}} \int_{\frac{l-w}{2}-y}^{\frac{w-l}{2}+y} -y J_C dy dx + \int_{-\frac{w-l}{2}}^{\frac{w-l}{2}} \int_{\frac{l-w}{2}+y}^{\frac{w-l}{2}-y} y J_C dy dx \right]. \quad (3)$$

From this, the critical current can be calculated from the magnitude of a DCM hysteresis loop,

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$$\Delta M = \frac{J_C \left( 3w - \frac{w^2}{l} \right)}{6}. \quad (4)$$

### B. Ac Susceptibility

For a rectangular superconducting film in the critical state in a sinusoidally oscillating field  $B = B_a \sin(\omega t)$ , the real ( $\chi'$ ) and imaginary ( $\chi''$ ) susceptibilities have been calculated analytically [4, 5] using

$$\begin{aligned} \chi'(\gamma) &= \frac{2\chi_0}{\pi} \int_0^\pi \left( 1 - \cos \theta \right) S \left( \frac{(1 - \cos \theta)\gamma}{2} \right) \cos \theta d\theta \end{aligned} \quad (5)$$

and

$$\begin{aligned} \chi''(\gamma) &= \frac{2\chi_0}{\pi} \int_0^\pi \left[ \left( 1 - \cos \theta \right) S \left( \frac{(1 - \cos \theta)\gamma}{2} \right) - S(\gamma) \right] \sin \theta d\theta. \end{aligned} \quad (6)$$

where  $\gamma$  is the field penetration parameter, the fractional distance into the sample which the field can penetrate,

$$\gamma = \frac{3\pi\chi_0 B_a}{\left( 3w - \frac{w^2}{l} \right) \mu_0 J_C} \quad (7)$$

and

$$S(x) = \frac{1}{2x} \left[ \arccos \left( \frac{1}{\cosh x} \right) + \frac{\tanh x}{\cosh x} \right]. \quad (8)$$

$\chi_0$  is the susceptibility at zero field, which has been found numerically using values of  $\beta = \frac{l}{w}$  and  $\zeta = \frac{t}{w}$  for our tape geometry and is well fitted by the formula [6],

$$\begin{aligned} \zeta\chi_0(\beta) &= \arctan[1 - 0.7223\beta^{-0.954} \\ &\quad + 0.3522\beta^{-2.57} \\ &\quad - 0.141\beta^{-3.66}]. \end{aligned} \quad (9)$$

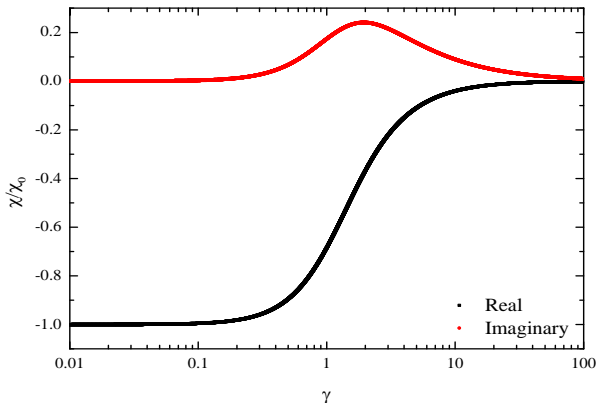


Fig 1: Normalised susceptibility vs penetration parameter calculated numerically.

The experimental harmonic susceptibility data were used to find the magnetic moment,  $m_{ac}$ , induced by the applied field, given by

$$m_{ac}(t) = V \times H_a \sum_{n=1}^{\infty} \chi'_n \sin(n\omega t) + \chi''_n \cos(n\omega t), \quad (10)$$

from which measured susceptibilities could be compared to the analytical expressions (Fig 1) and a value of  $\gamma$  (and hence  $J_C$ ) could be determined.

### III. EXPERIMENTAL RESULTS

Measurements were conducted using a Quantum Design PPMS in fields up to 8.5 T over the temperature range 4.2 → 95 K on a 4 mm × 5.5 mm × 1.6 μm sample of GdBCO HTS tape provided by SuNAM. The tape was oriented such that the applied field was perpendicular to the tape surface and parallel to the crystallographic c-axis.

Initially, the field was swept to −1.5 T to establish the ‘roof top’ internal field profile. Field sweeps were then carried out from −1.5 → 8.5 → −1.5 T and Dc magnetisation and susceptibility measurements were performed. The process was

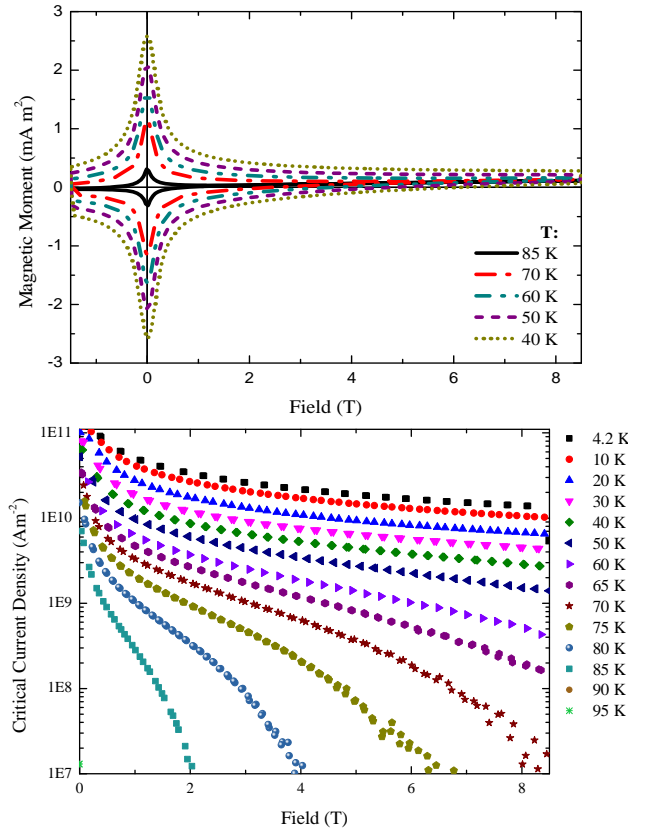


Fig. 2: (a) Dc Magnetisation loops of GdBCO and (b)  $J_C$  calculated using Bean's model from the magnetisation data.

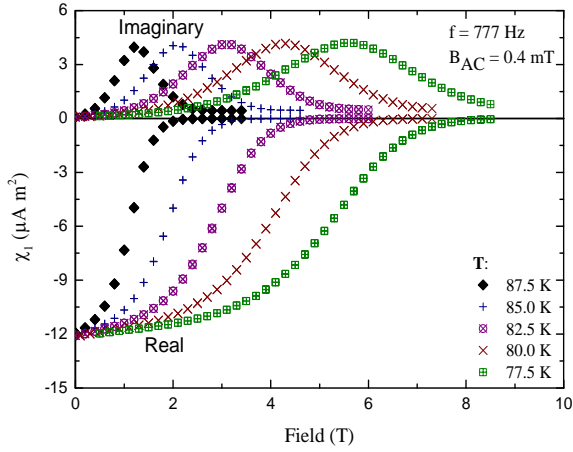


Fig. 3: Experimental values for susceptibility (1f) as a function of Dc field at various temperatures.

repeated at 10 K intervals for  $T < 60$  K and 5 K intervals for  $T > 60$  K. The ACS measurements in the PPMS were conducted in the 4-point measurement mode.

#### A. Dc Magnetisation

The Dc magnetization loops, shown in Fig. 2(a), were used to calculate  $J_C$  (Fig. 2(b)) as a function of applied field and temperature using Eq. (4). These data are consistent with the critical current density expected for this SuNAM tape,  $J_C = 2.2 \times 10^{10} \text{ Am}^{-2}$  at 77 K and 0 T. The field was swept at a rate of  $20 \text{ mTs}^{-1}$ . Using Faraday's law, this is equivalent to an electric field in the sample of  $E \approx 20 \text{ } \mu\text{Vm}^{-1}$ .

#### B. Ac Susceptibility

Harmonics of the susceptibility were measured up to the 10<sup>th</sup> multiple of the excitation frequency, at excitation frequencies of 77, 388, 777, 3888 and 7777 Hz. The magnitude of the excitation field,  $B_{ac,0}$ , was varied between 0.2 and 1 mT.

The measured harmonics,  $\chi'_n$  and  $\chi''_n$  (Fig. 5) were combined using Eq. (10) to find the magnetic moment,  $m_{ac}(t)$  induced by the applied field (Fig 4). The moment observed

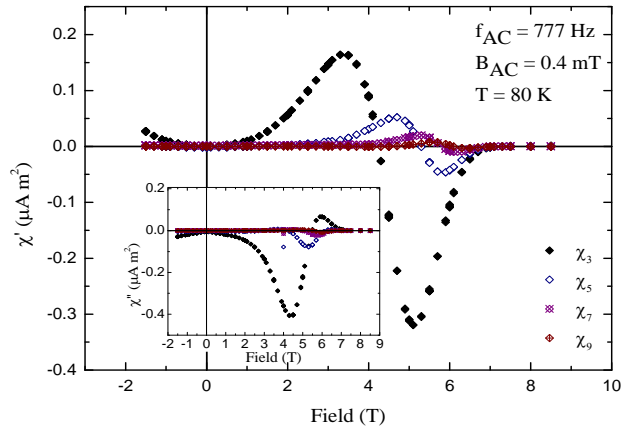


Fig. 5: Experimental harmonics of susceptibility,  $\chi'$  and (inset)  $\chi''$ .

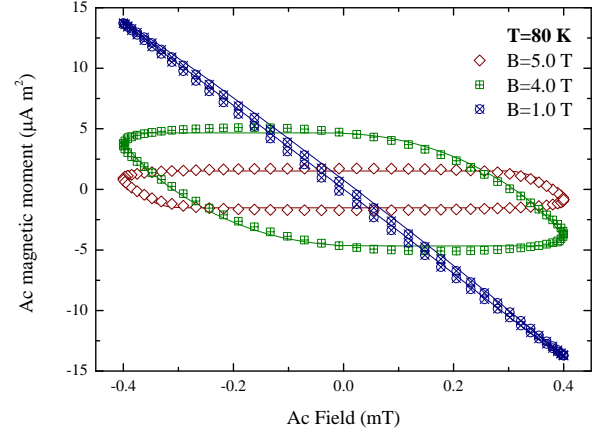


Fig 4: Comparison between moment,  $m_{ac}$ , induced by an applied field (points) and numerical calculations (line) at various values of applied Dc field at 80 K.

experimentally was fitted using Eqs. (5), (6) to find a value for the penetration parameter,  $\gamma$ , from which  $J_C$  was calculated. We note that for  $\gamma < 1$  we used the fit to find  $\frac{1}{\gamma}$  and for  $\gamma > 1$  we used the fit to find  $\gamma$ . In the cross-over region  $\gamma \cong 1$ , the fitting algorithms introduced errors in measured  $J_C$ . Data from this region ( $J_C \approx 4 \times 10^9 \text{ Am}^{-2}$ ) has been omitted from Fig 7.

$\gamma$  is the fractional distance into the sample which the applied field is able to penetrate. For  $\gamma < 1$ , the field is not sufficient to fully penetrate the sample and  $m_{ac}$  exhibits only small hysteretic effects. For  $\gamma > 1$ , the field fully penetrates the sample at a fractional time  $\tau_\gamma = \frac{1}{\pi} \cos^{-1}(1 - \frac{2}{\gamma})$  during the field cycle. For  $\tau_\gamma \ll 1$ ,  $m_{ac}$  becomes very small. The most accurate values for  $J_C$  are obtained when  $\gamma = \tau_\gamma = 1$ .

In Fig 4 we show the moment over the course of an field cycle for  $\gamma = 0.11, 1.57, 4.84$  at  $B = 1.0, 4.0, 5.0 \text{ T}$  respectively. Fig 6 shows  $J_C$  calculated by ACS and DCM techniques at different applied fields and temperatures. **Due to inaccuracies in the fitting program, we have chosen to omit some data in Figs 6, and 7 around  $\gamma = 1$ .**  $J_C$  calculated from

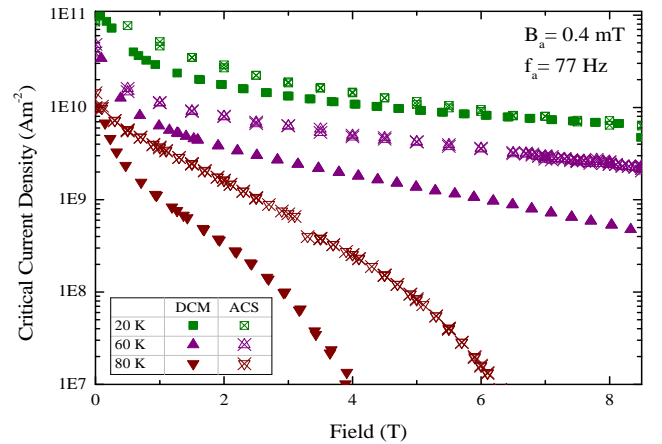


Fig 6:  $J_C$  measured using DCM (solid) and using ACS (x-centre) techniques up to 8.5 T at 20, 60 and 80 K.

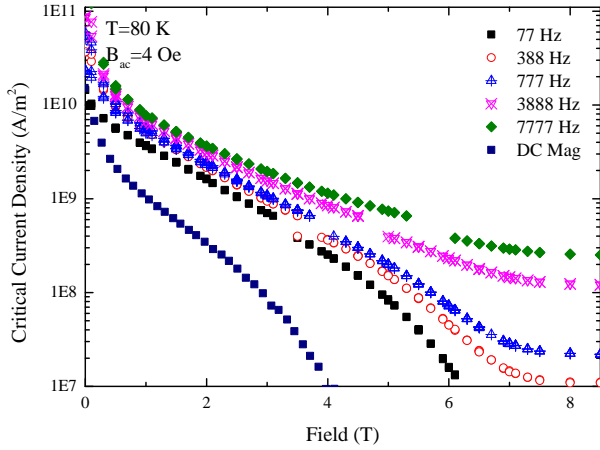


Fig 7: Comparison between  $J_C$  measured at different applied field frequencies with DCM data.

ACS measurements agree with DCM measurements at low  $\gamma$ , when  $J_C$  is large and the Dc field inhomogeneity is small compared to the self-field of the sample. At higher  $\gamma$ , there is a systematic underestimate of  $J_C$ .

The index of transition ( $n$ ) is defined through the relation

$$E = E_C \left( \frac{J}{J_C} \right)^n. \quad (11)$$

Bean's model effectively assumes  $n = \infty$  which means the electric field in the superconductor is either zero,  $E_C$  or infinity. However, we can use Faraday's law to estimate the RMS electric field strength within the sample during measurements

$$E_{rms} \approx \begin{cases} \frac{B_a \omega}{\sqrt{2}} \times \frac{wl}{2(w+l)} \times \gamma & \gamma < 1 \\ \frac{B_a \omega}{\sqrt{2}} \times \frac{wl}{2(w+l)} & \gamma \geq 1. \end{cases} \quad (12)$$

The rms E-field at a field magnitude of 0.4 mT and a frequency of 77 Hz is  $E_{rms} \approx 20 \text{ mVm}^{-1}$ . This value is similar to the E-field in our Dc measurements. Using the variable frequency data in Fig. 7, values of  $E_{rms}$  and  $J$  were calculated and plotted in Fig. 8.

The values of the index of transition,  $n$ , have been calculated from the  $E_{rms}$  versus  $J$  in Fig 8 and are shown in Table 1. These are consistent with expected values at 80 K [7]. It is important to note that one can only expect  $J_C$  measurements made by Ac and Dc methods to agree when the electric fields induced in the two measurements are similar.

TABLE I  
VALUES OF THE INDEX OF TRANSITION CALCULATED FROM ACS  
MEASUREMENTS AT 80 K

$B$	1.0 T	3.0 T	5.0 T	6.0 T	7.0 T
$n$	6.5	4.5	2.1	1.4	1.0

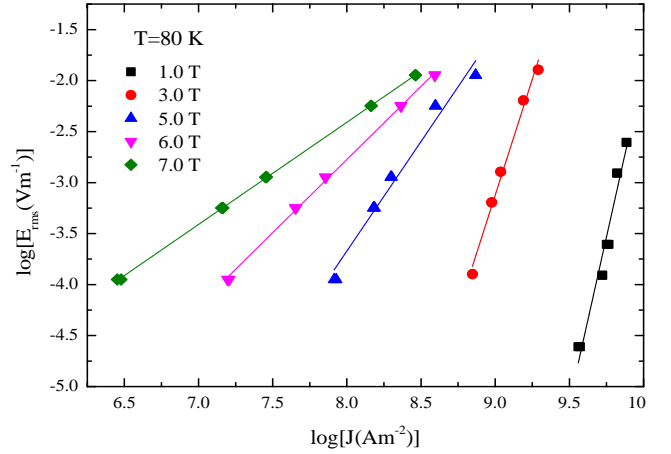


Fig 8: Log-log plot of internal rms electric field,  $E_{rms}$  and current density,  $J$ . The gradient of each line provides the index of transition at each magnetic field.

#### IV. DISCUSSION

DCM and ACS measurement techniques have been directly compared using a sample of superconducting GdBCO tape. The error in  $J_C$  measured by DCM techniques grows as  $J_C$  decreases.

This is attributed to the error caused by Dc field inhomogeneity in the DCM measurement. We note that the harmonic analysis used in this paper could be used to correct Vibrating Sample Magnetometry measurements for any applied field inhomogeneity by reconstructing the total time dependent moment  $m(t)$  from the voltage signal in the pick-up coils [1].

A simple Bean's model analysis of ACS data has been used to measure  $J_C$  for varying excitation fields. We have used these data to obtain estimates for the index of transition,  $n$ , at 80 K over the entire field range. In future, ACS data could be run with varying excitation fields and the  $J_C$  data extrapolated back to any arbitrary E-field criterion. It is also possible to further refine ACS measurements by considering the internal electric field using Brandt's model for rectangular thin films [8, 9] instead of Bean's analysis.

#### V. CONCLUSION

We have measured the critical current density  $J_C$  of a thin film GdBCO sample using ACS and DCM measurements. The error in DCM measurements has been attributed to inhomogeneity in the applied Dc field and is characterized by the penetration parameter,  $\gamma$ . Using a harmonic analysis of ACS data, we have obtained more accurate measurements of  $J_C$  for  $\gamma \approx 1$ . The index of transition,  $n$ , has also been calculated for various applied Dc fields at 80 K and could, in principle, be used to extract  $J_C$  from ACS data at any arbitrary field criterion.

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